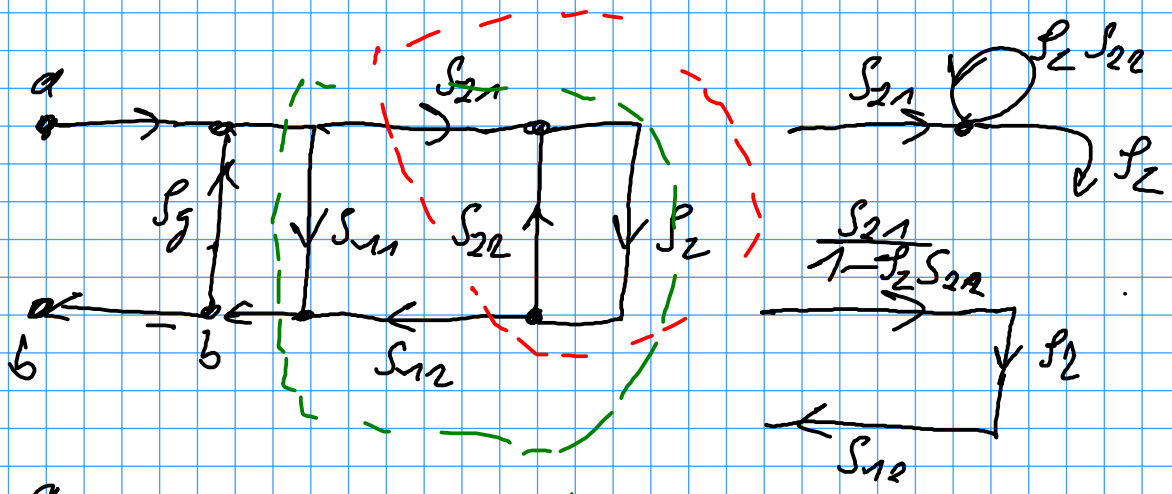
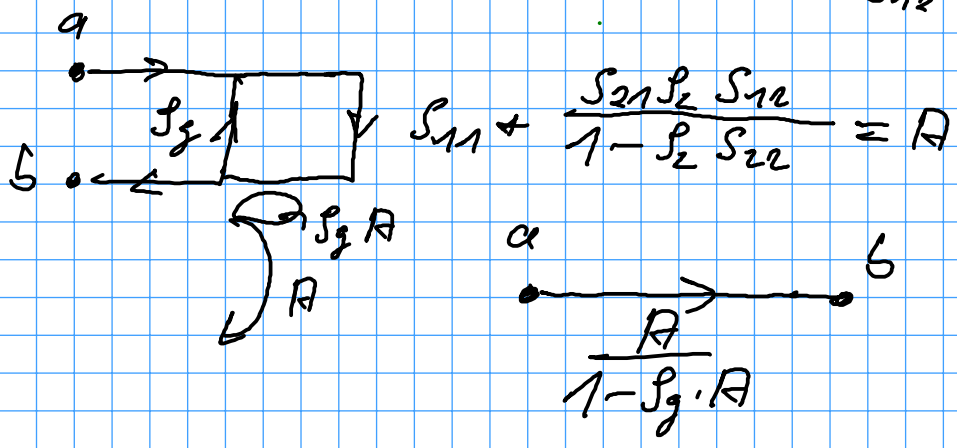
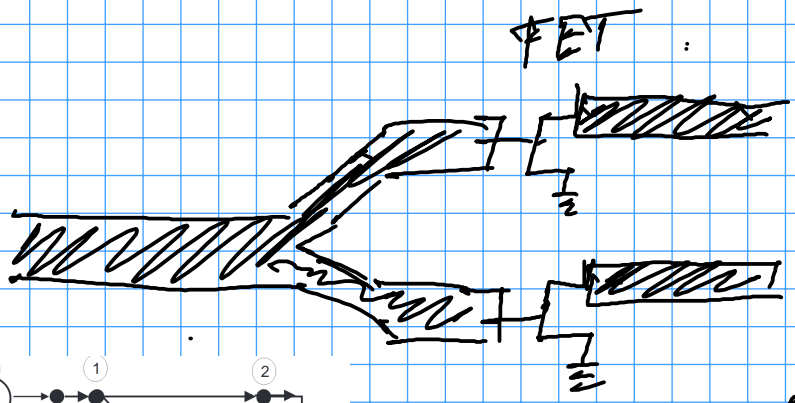
1a)

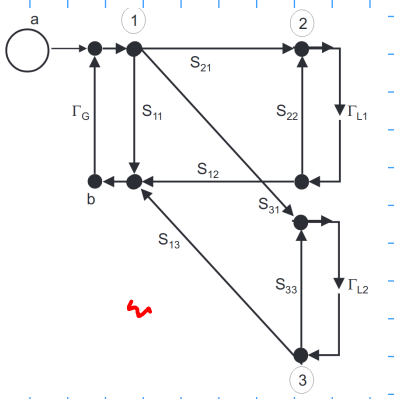


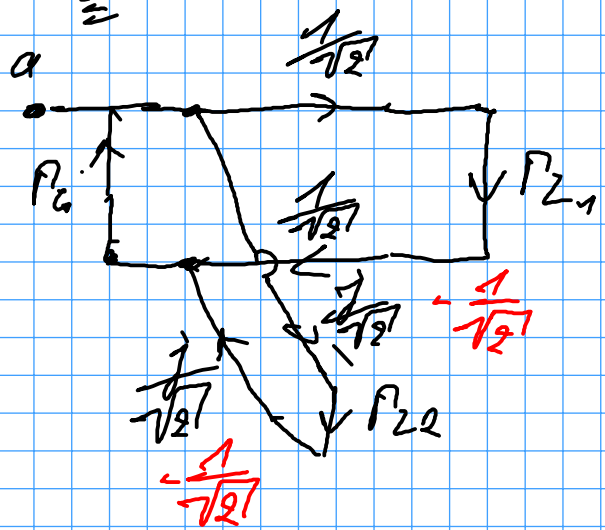


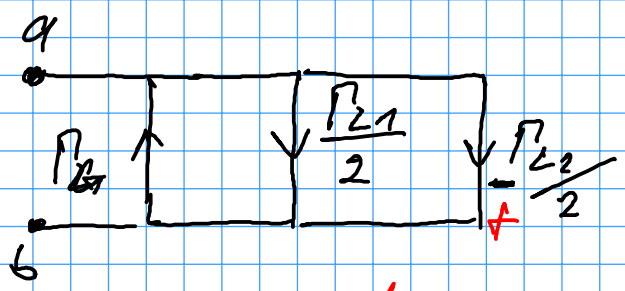
$$\frac{b}{a} = \frac{S_{11} - \rho_L S_{11} S_{22} + S_{21}\rho_L S_{12}}{1 - \rho_L \cdot S_{22} - \rho_g \cdot S_{11} + \rho_g \rho_L S_{11} S_{22} - \rho_g \rho_L S_{12} S_{21}}$$

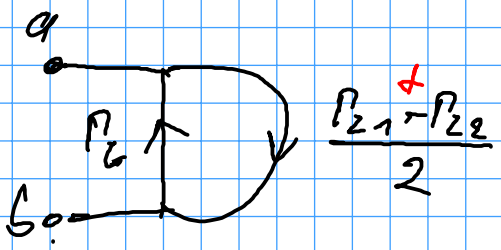
2)







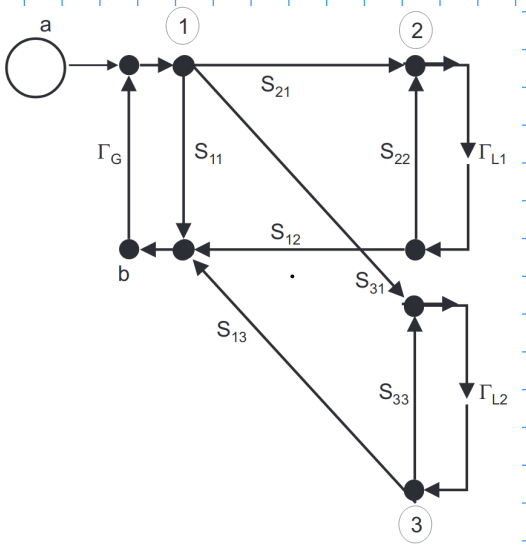


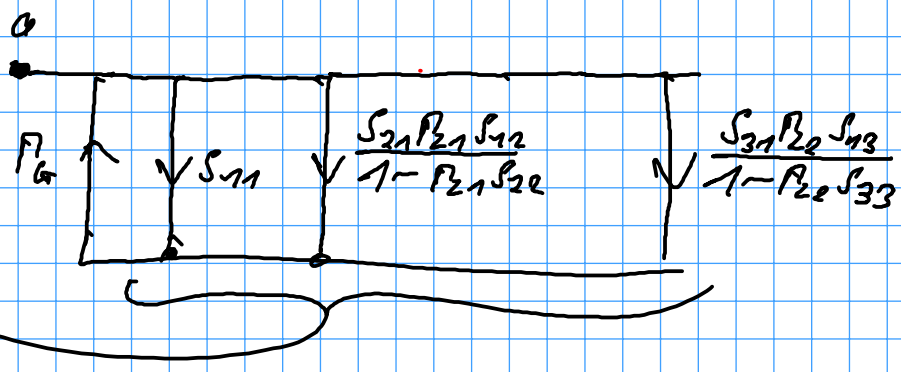


$$\frac{b}{a} = \frac{\frac{\Gamma_{L1} - \Gamma_{L2}}{2}}{1 - \Gamma_G \cdot \left(\frac{\Gamma_{L1} - \Gamma_{L2}}{2}\right)}$$

not für Aufgabenteil b

c)





$$\frac{S_{11}\left(1-\Gamma_{L1}S_{22}\right)\left(1-\Gamma_{L2}S_{33}\right) + S_{21}\Gamma_{L1}S_{12}\left(1-\Gamma_{L2}S_{33}\right) + S_{31}\Gamma_{L2}S_{13}\left(1-\Gamma_{L1}S_{22}\right)}{\left(1-\Gamma_{L1}S_{22}\right)\left(1-\Gamma_{L2}S_{33}\right)}$$

$$\frac{b}{a} = \frac{S_{11}\left(1-\Gamma_{L1}S_{22}\right)\left(1-\Gamma_{L2}S_{33}\right) + S_{21}\Gamma_{L1}S_{12}\left(1-\Gamma_{L2}S_{33}\right) + S_{31}\Gamma_{L2}S_{13}\left(1-\Gamma_{L1}S_{22}\right)}{\left(1-\Gamma_{L1}S_{22}\right)\left(1-\Gamma_{L2}S_{33}\right) - \Gamma_G S_{11}\left(1-\Gamma_{L1}S_{22}\right)\left(1-\Gamma_{L2}S_{33}\right) - \Gamma_G S_{21}\Gamma_{L1}S_{12}\left(1-\Gamma_{L2}S_{33}\right) - \Gamma_G S_{31}\Gamma_{L2}S_{13}\left(1-\Gamma_{L2}S_{22}\right)}$$

d) Man verwendet Variante (a) mit 90° Koppler, weil hier der Differenz der Einzelreflexionen auftritt.

e) $r_{L1} = 1$. $r_{L2} = 0{,}316$ $r_G = 0{,}1$

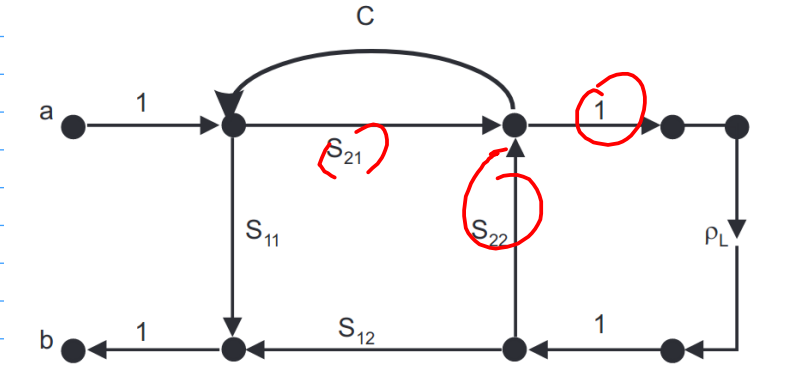
a) $\frac{b}{a} = \frac{1 - 0{,}316}{2 - 0{,}1(1 - 0{,}316)} = \underline{\underline{0{,}354}}$

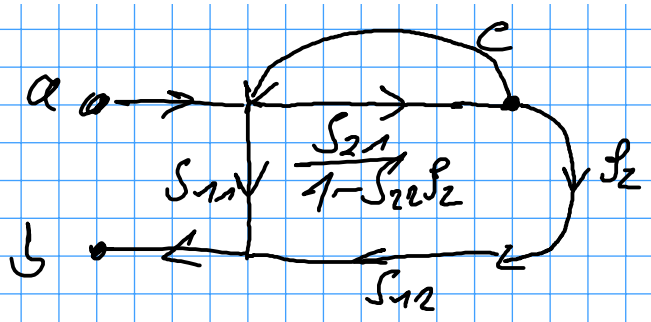
b) $\frac{b}{a} = \frac{1 + 0{,}316}{2 - 0{,}1(1 + 0{,}316)} = \underline{\underline{0{,}704}}$

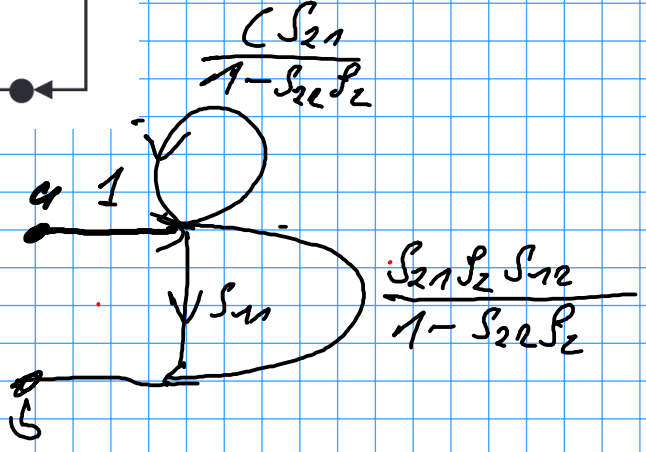
3a)

$$\frac{b}{a} = S_{11} + \frac{S_{21} \rho_2 S_{12}}{1 - \rho_2 S_{22}}$$

b)







$$\frac{b}{a} = \frac{1}{1 - \frac{C S_{21}}{1 - S_{22}\rho_2}} \cdot \left(S_{11} + \frac{S_{21}\rho_2 S_{12}}{1 - S_{22}\rho_2} \right)$$

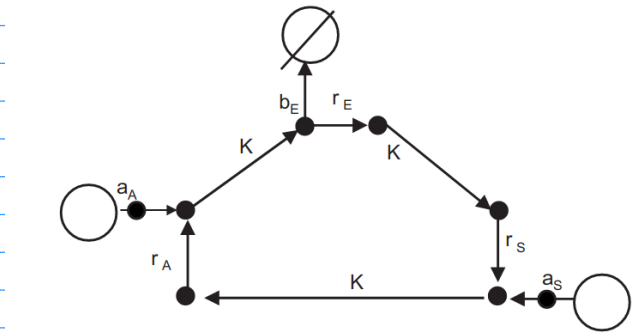
$$= \frac{1}{1 - S_{22}\rho_2 - C S_{21}} \cdot \left(S_{11} - S_{11} S_{22} \rho_2 + S_{21}\rho_2 S_{12} \right)$$

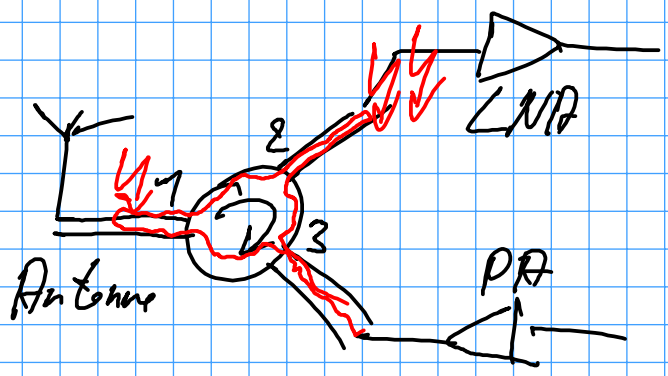
Rückkopplung soll zu Null werden

c)

$$S_{22}\rho_2 + C S_{21} = 0 \quad \Leftrightarrow \quad C = -\frac{S_{22}\rho_2}{S_{21}}$$

mit $|S_{22}|, |\rho_2| < 1$ und $|S_{21}| > 1$ ist C passiv realisierbar.





Zirkulator Streumatrix

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & s_{13} \\ s_{21} & 0 & 0 \\ 0 & s_{32} & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$b_E = K \cdot a_A + K^2 \cdot r_A \cdot a_S + K^3 \cdot r_E \cdot r_S \cdot r_A \cdot b_E$$

$$\Leftrightarrow b_E = \frac{K a_A + K^2 r_A a_S}{1 - K^3 r_E r_S r_A}$$

Mit $r_A = 0$ liegt eine vollständige Entkopplung vor.

4) a/b

$$\frac{b}{a} = S_{11} + \frac{S_{12} S_{21} \rho_2}{1 - S_{22} \rho_2}$$

$$0{,}5 = 0{,}2 + \frac{0{,}64\, \rho_2}{1 - 0{,}2\, \rho_2}$$

$$\Leftrightarrow \quad 0{,}5 - 0{,}1\rho_2 = 0{,}2 - 0{,}04\rho_2 + 0{,}64\rho_2$$

$$\Leftrightarrow \quad 0{,}3 = 0{,}7\rho_2 \quad \Leftrightarrow \quad \rho_2 = \frac{3}{7}$$

c)

bei $\rho_L = 0$: $\frac{b}{a} = 0.1$

bei $\rho_L = 1$: $\frac{b}{a} = 1$

bei $\rho_L = -1$: $\frac{b}{a} = -\frac{7}{11}$

$$\frac{b}{a} = S_{11} + \frac{S_{12} S_{21} \rho_2}{1 - S_{22} \rho_2}$$

$\rho_2 = 0$; $0{,}1 = S_{11}$

$\rho_2 = 1$; $1 = S_{11} + \frac{S_{12} S_{21}}{1 - S_{22}}$

$\rho_2 = -1$; $-\frac{7}{11} = S_{11} - \frac{S_{12} S_{21}}{1 + S_{22}}$

d) $S_{11} = 0{,}1$

e) $S_{22} = 0$ $\quad 1 = 0{,}1 + S_{12} S_{21} \quad \Leftrightarrow \quad S_{12} = -\sqrt{0{,}9} = 0{,}949$

f)

$$0{,}9 = \frac{S_{12} S_{21}}{1 - S_{22}}$$

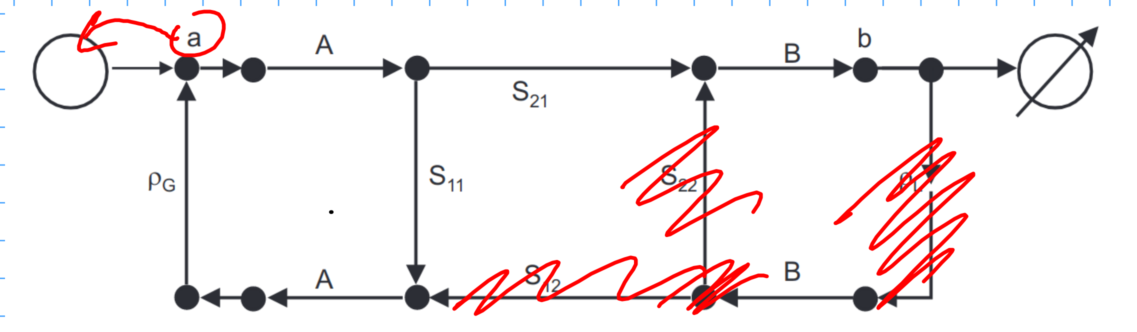
$$-\frac{7}{11} - 0{,}1 = \frac{-S_{12} S_{21}}{1 + S_{22}} \qquad / \; \frac{-0{,}9}{-\frac{8}{11} - 0{,}1}$$

$$= \frac{1 + S_{22}}{1 - S_{22}} = A = + \frac{11}{9}$$

$$1 + S_{22} = A - A S_{22} \quad \Leftrightarrow \quad S_{22} = \frac{1 - A}{1 + A}(-1) = 0{,}1$$

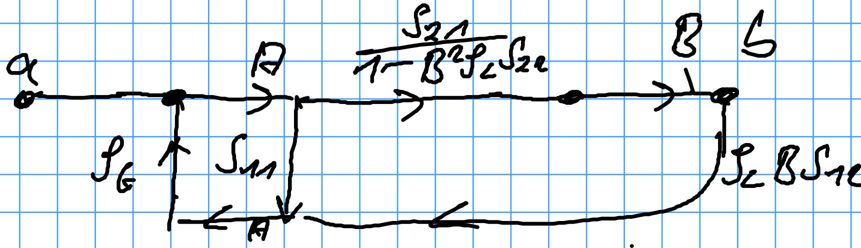
$$S_{12} = -\sqrt{0{,}9 \cdot (1 - S_{22})} = 0{,}9$$

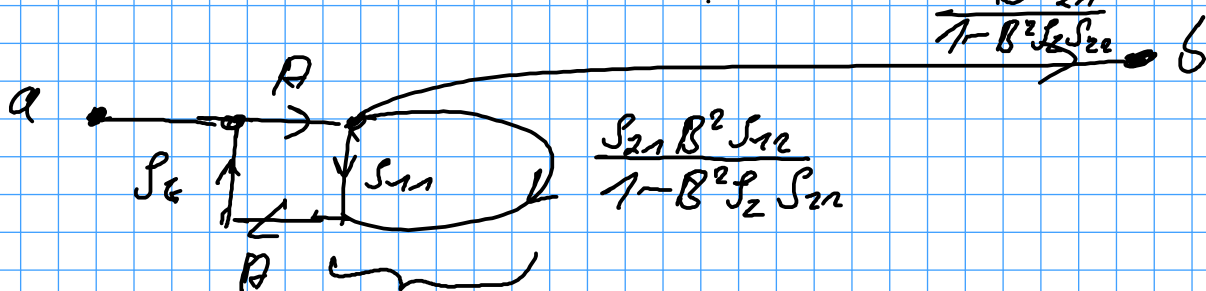
5a)



$$\frac{b}{a} = \frac{A B S_{21}}{1 - A^2 \rho_G S_{11}}$$

b)





$$\frac{S_{11} - B^2 \rho_L S_{11} S_{22} + S_{21} B^2 S_{12}}{1 - B^2 \rho_L S_{22}}$$

$$\frac{b}{a} = \frac{A B S_{21}}{1 - B^2 \rho_L S_{22} - A^2 \rho_G \left(S_{11} + B^2 \rho_L S_{11} S_{22} + S_{21} B^2 S_{12}\right)}$$

c)

$$\frac{b}{a} = \frac{A B S_{21}}{1 - A^2 \rho_G S_{11}}$$

$$\Leftrightarrow S_{21} = \frac{\frac{b}{a} \cdot \left(1 - A^2 \rho_G S_{11}\right)}{AB}$$

d) $|S_{11}| = 1$ worst Case! $|A|, |B| = 1$

$\rho_G = 0{,}316$: $\frac{b}{a} \cdot (1 - 0{,}316) \le S_{21} \le \frac{b}{a}(1 + 0{,}316)$ $\pm 30\%$

$\rho_G = 0{,}1$: $\frac{b}{a}(1 - 0{,}1) \le S_{21} \le \frac{b}{a}(1 + 0{,}1)$ $\pm 10\%$