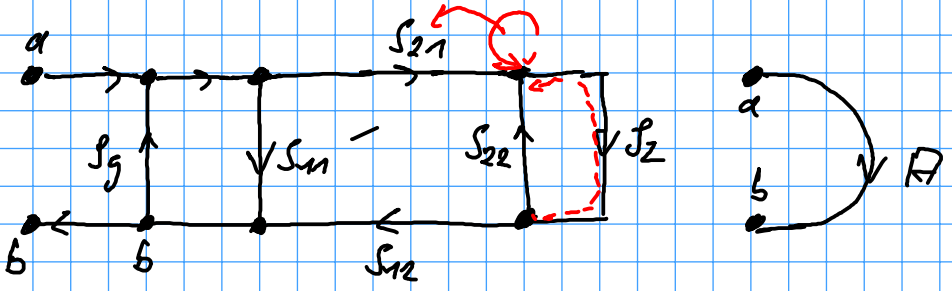
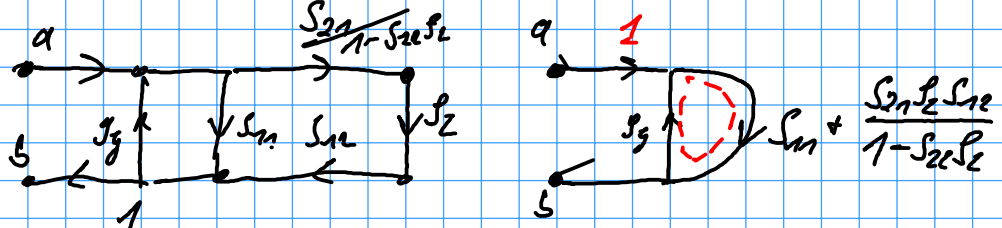
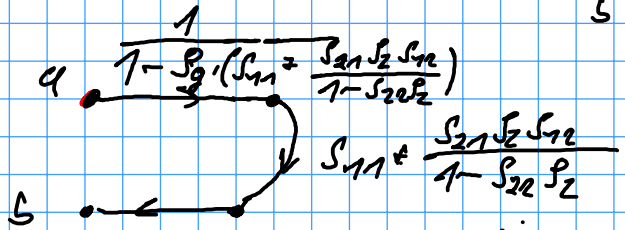
Signalflussgraphen

1a)







$$\frac{b}{a} = \frac{S_{11} + \frac{S_{21}S_Z S_{12}}{1-S_{22}S_Z}}{1 - S_g\left(S_{11} + \frac{S_{21}S_Z S_{12}}{1-S_{22}S_Z}\right)}$$

$$= \frac{S_{11}(1-S_{22}S_Z) + S_{21}S_Z S_{12}}{1 - S_{22}S_Z - S_g S_{11}\cdot(1-S_{22}S_Z) - S_g S_{21}S_Z S_{12}}$$

b) $$\frac{b}{a} = \frac{S_{21}S_Z S_{12}}{1 - S_g S_{21}S_Z S_{12}}$$

$$\frac{b}{a} = S_{21}S_Z S_{12} + \frac{b}{a}\cdot S_g S_{21}S_Z S_{12} = S_Z\left(S_{21}S_{12} + \frac{b}{a}S_g S_{21}S_{12}\right)$$

$$\Rightarrow S_Z = \frac{\frac{b}{a}}{S_{21}S_{12}\cdot\left(1 + \frac{b}{a}S_g\right)}$$

c) $$S_Z = \frac{A\frac{b}{a} + B}{C\cdot\frac{b}{a} + D}$$

$A = 1$, $B = 0$

$C = S_{21}S_{12}S_g$; $D = S_{21}S_{12}$

Wird zur Fehlerkorrektur bspw. bei elektrischen Netzwerkanalysatoren verwendet.

d) $$\frac{b}{a} = \frac{S_{11}(1 - S_{22}\Gamma_2) + S_{21}\Gamma_2 S_{12}}{1 - S_{22}\Gamma_2 - \Gamma_g S_{11}\cdot(1 - S_{22}\Gamma_2) - \Gamma_g S_{21}\Gamma_2 S_{12}}$$

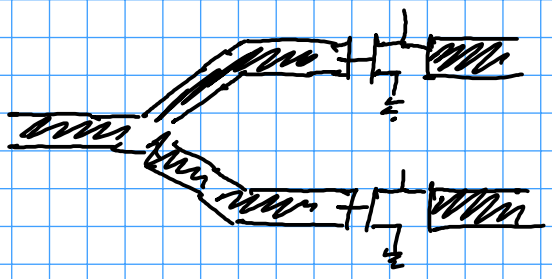
mit $\Gamma_g = 0$

$$\frac{b}{a} = \frac{S_{11} - S_{22}S_{11}\Gamma_2 + S_{21}\Gamma_2 S_{12}}{1 - S_{22}\Gamma_2}$$

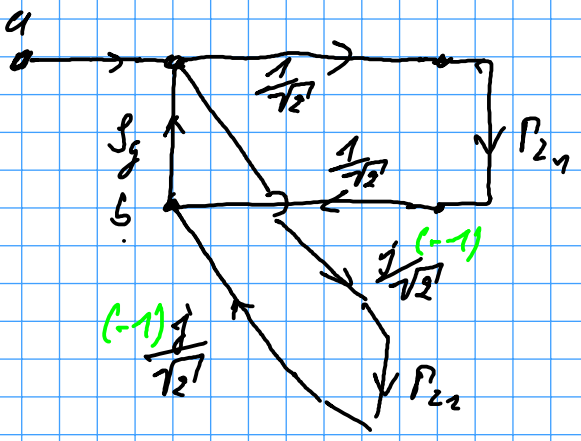
$$\Leftrightarrow \frac{b}{a} - S_{11} = \Gamma_2 \cdot \left(-S_{11}S_{22} + S_{21}S_{12} + S_{22}\frac{b}{a}\right)$$

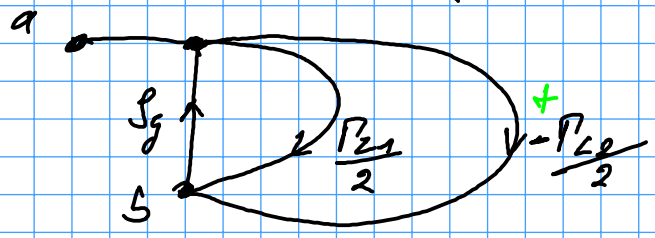
$$\Gamma_2 = \frac{\frac{b}{a} - S_{11}}{\frac{b}{a}S_{22} + S_{21}S_{12} - S_{11}S_{22}}$$

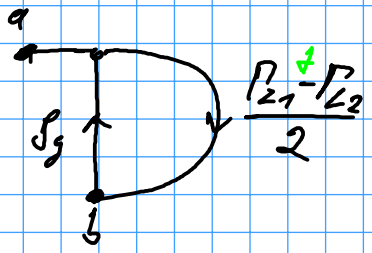
2)

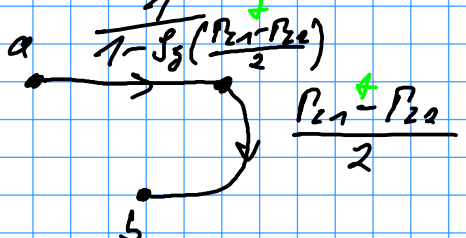


a)





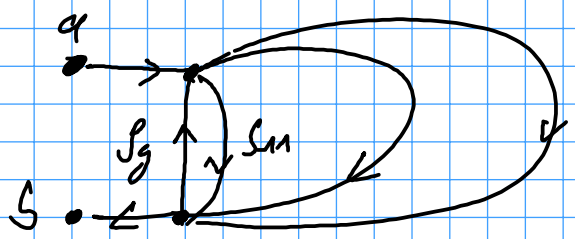




$$\frac{b}{a} = \frac{\Gamma_{L1} - \Gamma_{L2}}{2 - \Gamma_g(\Gamma_{L1} - \Gamma_{L2})}$$

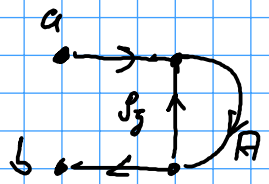
b) $$\frac{b}{a} = \frac{\Gamma_{z1} + \Gamma_{z2}}{2 - s_g(\Gamma_{z1} + \Gamma_{z2})}$$

c)



$$\frac{s_{31}\Gamma_{z2}s_{13}}{1 - s_{33}\Gamma_{z2}}$$

$$\frac{s_{21}\Gamma_{z1}s_{12}}{1 - s_{22}\Gamma_{z1}}$$



$$s_{11} + \frac{s_{21}\Gamma_{z1}s_{12}}{1 - s_{22}\Gamma_{z1}} + \frac{s_{31}\Gamma_{z2}s_{13}}{1 - s_{33}\Gamma_{z2}}$$

$$= \frac{s_{11}(1 - s_{22}\Gamma_{z1})(1 - s_{33}\Gamma_{z2}) + s_{21}s_{12}\Gamma_{z1}(1 - s_{33}\Gamma_{z2}) + s_{31}\Gamma_{z2}s_{13}(1 - s_{22}\Gamma_{z1})}{(1 - s_{22}\Gamma_{z1})(1 - s_{33}\Gamma_{z2})} = A$$

$$\frac{b}{a} = \frac{A}{1 - s_g \cdot A} = \frac{s_{11}(1 - s_{22}\Gamma_{z1})(1 - s_{33}\Gamma_{z2}) + s_{21}s_{12}\Gamma_{z1}(1 - s_{33}\Gamma_{z2}) + s_{31}\Gamma_{z2}s_{13}(1 - s_{22}\Gamma_{z1})}{(1 - s_{22}\Gamma_{z1})(1 - s_{33}\Gamma_{z2}) - s_g[s_{11}(1 - s_{22}\Gamma_{z1})(1 - s_{33}\Gamma_{z2}) + s_{21}s_{12}\Gamma_{z1}(1 - s_{33}\Gamma_{z2}) + s_{31}\Gamma_{z2}s_{13}(1 - s_{22}\Gamma_{z1})]}$$

d) Variante a ist zu bevorzugen, weil die Reflexion $\frac{b}{a}$ minimal wird bei $\Gamma_{z1} = \Gamma_{z2}$ sogar verschwindet.

e) $\Gamma_{z1} = 1$;

$|\Gamma_{L2}| = -10\,dB$ $\quad \Gamma_{L2} = \frac{1}{\sqrt{10}} \approx 0{,}316$

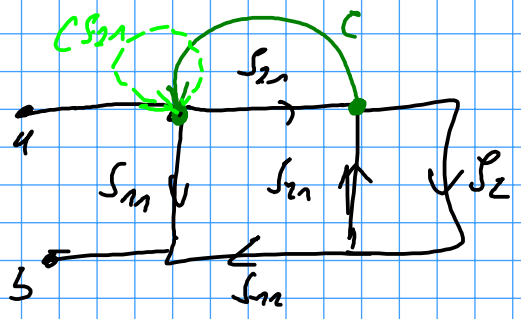
$|\Gamma_G| = -20\,dB$ $\quad \Gamma_G = \frac{1}{10} = 0{,}1 = s_g$

Reflexionsfaktor für Anpassung

a) $$\frac{b}{a} = \frac{\Gamma_{z1} - \Gamma_{z2}}{2 - s_g \cdot (\Gamma_{z1} - \Gamma_{z2})} = \frac{1 - 0{,}316}{2 - 0{,}1 \cdot (1 - 0{,}316)} = \underline{0{,}354} \triangleq -9\,dB$$

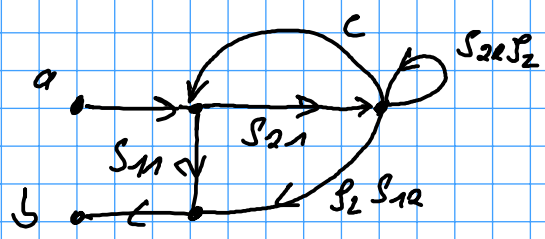
b) $$\frac{b}{a} = \frac{\Gamma_{z1} + \Gamma_{z2}}{2 - s_g(\Gamma_{z1} + \Gamma_{z2})} = \ldots = \underline{0{,}704} \triangleq -3\,dB$$

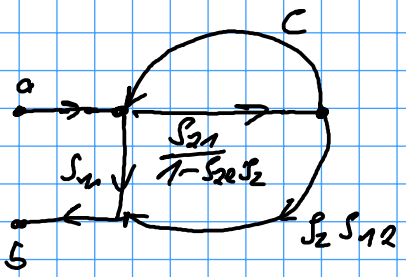
3 a)

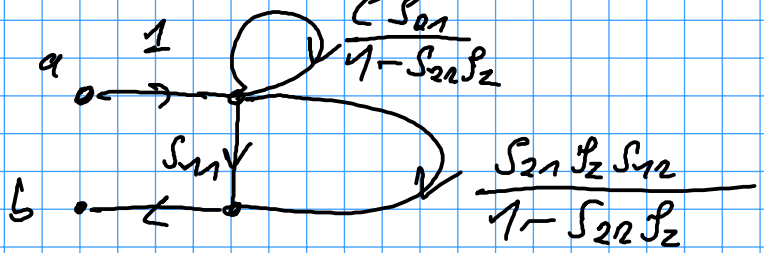


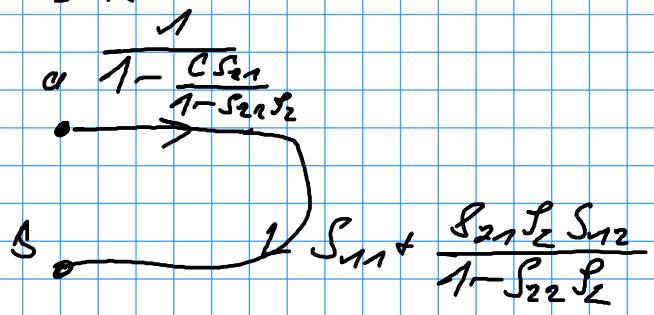
$$\frac{b}{a} = S_{11} + \frac{S_{21} S_Z S_{12}}{1 - S_{22} S_Z}$$

b)









$$\frac{b}{a} = \frac{S_{11}(1 - S_{22} S_Z) + S_{21} S_Z S_{12}}{1 - S_{22} S_Z - C S_{21}}$$

← das ist die Rückkopplung insgesamt.

c)

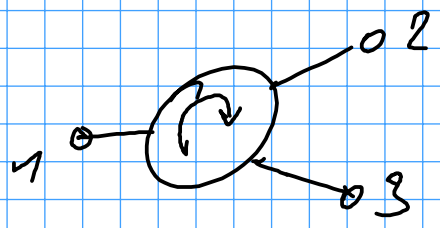
$$0 = -S_{22} S_Z - C S_{21} \quad \Leftrightarrow \quad C = \frac{-S_Z S_{22}}{S_{21}}$$

ist C passiv realisierbar ?

$|S_Z| < 1$, $|S_{22}| < 1$ $|S_{21}| > 1$ Verstärker / Transistor

$\Rightarrow |C| < 1$ also passiv realisierbar.

d)



ideal: Transmission von $1 \to 2$, aber nicht von $2 \to 1$
und $1 \to 3$
$2 \to 3$, $3 \to 1$

S-Parameter

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & S_{13} \\ S_{23} & 0 & 0 \\ 0 & S_{32} & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$b_E = K \cdot a_A + K^2 \cdot r_A \cdot a_S + K^3 \cdot r_E \cdot r_S \cdot r_A \cdot b_E$$

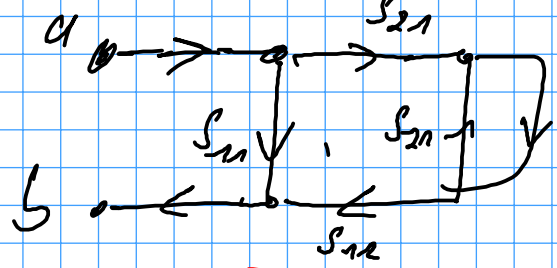
$$b_E = \frac{K a_A + K^2 r_A a_S}{1 - K^3 r_E r_A r_S}$$

e)

r_A muss optimal werden, wenn $r_A = 0$ ist, sind Sender und Empfänger ideal entkoppelt.
Die Koppelung entspricht $K^2 \cdot r_A$.

4)

a)



$$\frac{b}{a} = S_{11} + \frac{S_{21} S_L S_{12}}{1 - S_{22} S_L}$$

b)

$$0{,}5 = 0{,}2 + \frac{0{,}64 S_L}{1 - 0{,}2 S_L}$$

$$\Leftrightarrow 0{,}3 \cdot (1 - 0{,}2 S_L) = 0{,}64 S_L$$

$$\Leftrightarrow \quad 0{,}3 = 0{,}7 \cdot f_Z \quad \Leftrightarrow \quad f_Z = \frac{3}{7} = \underline{\underline{0{,}429}}$$

c) $$\frac{b}{a} = S_{11} + \frac{S_{21} S_{12} f_Z}{1 - S_{22} f_Z}$$

$f_Z = 0 \qquad \frac{b}{a} = S_{11} \quad \Rightarrow \quad 0{,}1 = S_{11}$

$f_Z = 1 \qquad \frac{b}{a} = S_{11} + \frac{S_{21} S_{12}}{1 - S_{22}} \qquad 1 = S_{11} + \frac{S_{21} S_{12}}{1 - S_{22}}$

$f_Z = -1 \qquad \frac{b}{a} = S_{11} - \frac{S_{21} S_{12}}{1 + S_{22}} \qquad -\frac{7}{11} = S_{11} - \frac{S_{21} S_{12}}{1 + S_{22}}$

d) $S_{11} = 0{,}1$

e) $S_{22} = 0$ mit $f_Z = 1 \qquad 1 = S_{11} + S_{12}^2$ —

$$\Leftrightarrow \quad S_{12} = -\sqrt{1 - S_{11}} = -\sqrt{0{,}9} = \underline{\underline{0{,}9487}}$$

f) $$1 - S_{11} = S_{21}^2 / 1 - S_{22}$$

$$-\frac{7}{11} - S_{11} = \frac{-S_{21}^2}{1 + S_{22}}$$

u. $$\frac{1 - S_{11}}{\underbrace{-\frac{7}{11} - S_{11}}_{A = -\frac{11}{9}}} = -\frac{1 + S_{22}}{1 - S_{22}}$$

$$\Leftrightarrow \quad A(1 - S_{22}) = -1 - S_{22}$$

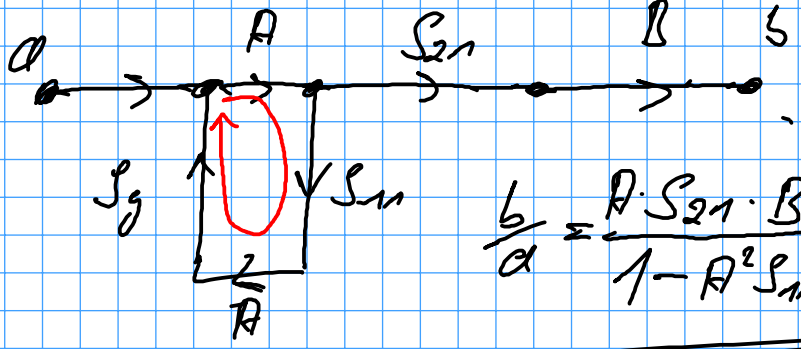
$$\Leftrightarrow \quad A + 1 = S_{22}(A - 1) \quad \Leftrightarrow \quad S_{22} = \frac{A + 1}{A - 1} = \underline{\underline{0{,}1}}$$

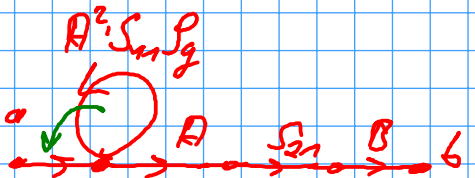
$$\rightarrow \quad S_{21}^2 = (1 - S_{11})(1 - S_{22}) = 0{,}81$$

$$\underline{\underline{S_{21} = 0{,}9}}$$

5)

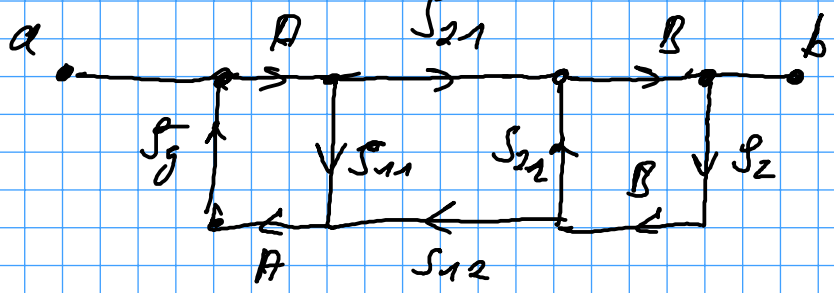
a)

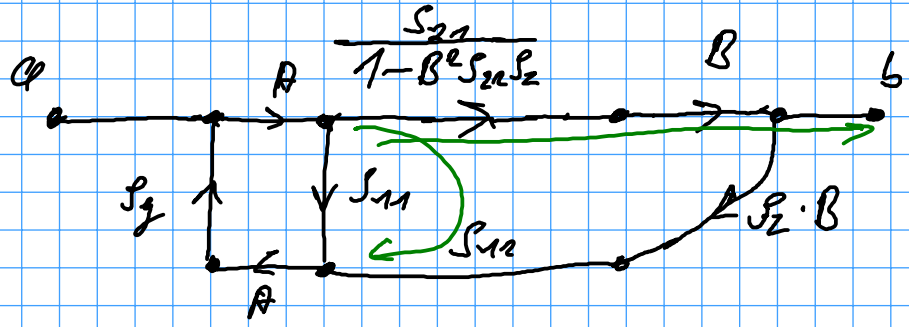


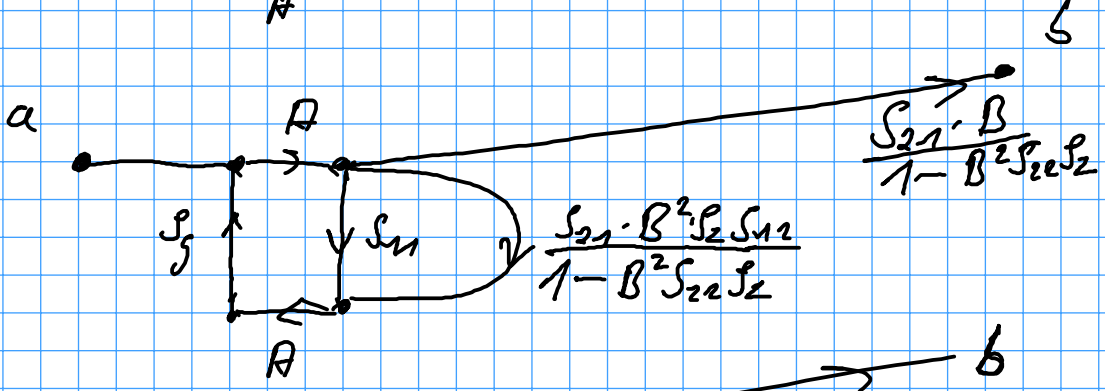


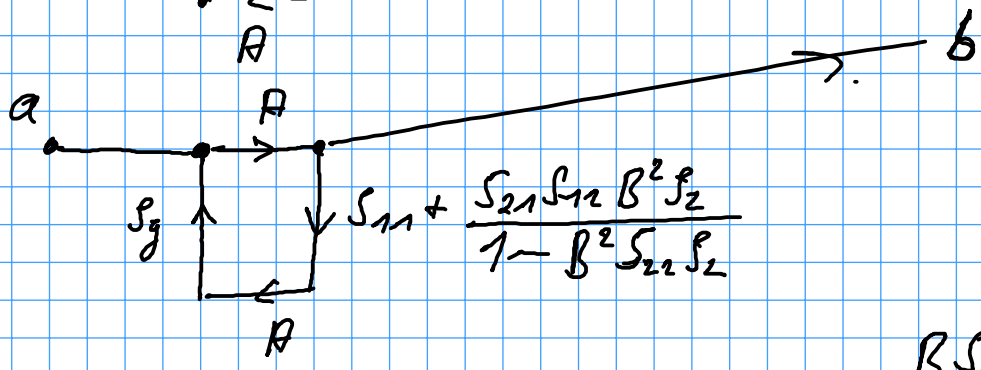
$$\frac{b}{a} = \frac{A \cdot S_{21} \cdot B}{1 - A^2 S_{11} S_g}$$

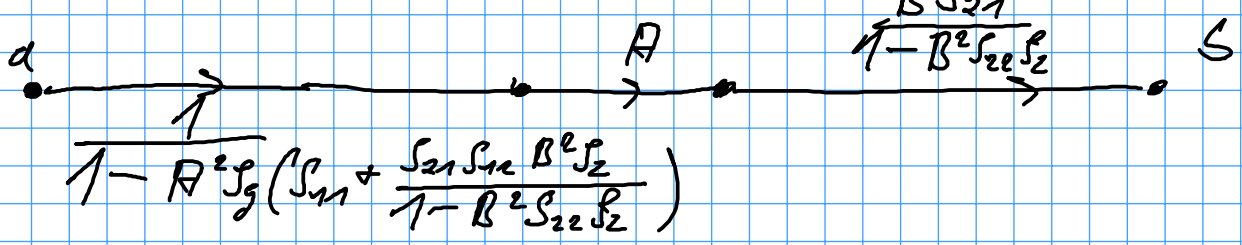
b)











$$\frac{b}{a} = \frac{A S_{21} B}{1 - B^2 S_{22} S_2 - A^2 S_g S_{11} + A^2 S_g S_{11} B^2 S_{22} S_2 - A^2 S_{21} S_{12} B^2 S_2 S_g}$$

c) $\frac{b}{a} = \frac{A \cdot S_{21} \cdot B}{1 - A^2 S_{11} S_g}$ Ergebnis von a

$$S_{21} = \frac{\frac{b}{a}(1 - A^2 S_{11} S_g)}{AB}$$

a) $S_g = -0{,}316$ oder $+0{,}316$

$$S_{21} = \frac{b/a}{AB} \cdot (1 - 0{,}316) \text{ oder } \frac{b/a}{AB}(1 + 0{,}316)$$

$$0{,}684 \cdot \frac{b/a}{AB} < |S_{21}| < 1{,}316 \frac{b/a}{AB}$$

Fehler ca $\pm 30\%$ in der Messung, wenn man S_g ignoriert

mit $S_g = \pm 0{,}1$ ist der Fehler dann $\pm 10\%$